I. GENERAL COURSE INFORMATION

Subject and Number: Mathematics 220
Descriptive Title: Multi-Variable Calculus
Course Disciplines: Mathematics
Division: Mathematical Sciences

Catalog Description:
This course is the study of calculus in several variables. Differentiation topics include partial differentiation, tangent planes to surfaces, directional derivatives and optimization. Integral topics include line, surface, and volume integrals, and the theorems of Green, Gauss and Stokes as generalizations of the Fundamental Theorem of Calculus. Topics in vector algebra and solid analytic geometry are included.

Conditions of Enrollment:
Prerequisite: Mathematics 191 with a minimum grade of C

Course Length:	**X Full Term**	**Other (Specify number of weeks):**
Hours Lecture:	**5.00 hours per week**	**TBA**
Hours Laboratory:	**0 hours per week**	**TBA**
Course Units:	**5.00**	

Grading Method: Letter
Credit Status: Associate Degree Credit

Transfer CSU: X Effective Date: Prior to July 1992
Transfer UC: X Effective Date: Prior to July 1992

General Education:
El Camino College:
4B – Language and Rationality – Communication and Analytical Thinking
Term: Other: Approved
6 – Mathematics Competency
Term: Other: Approved

CSU GE:
B4 - Mathematics/Quantitative Thinking
Term: Fall 2001 Other:

IGETC:
2A - Mathematical Concepts and Quantitative Reasoning
Term: Fall 2001 Other:

II. OUTCOMES AND OBJECTIVES

A. COURSE STUDENT LEARNING OUTCOMES (The course student learning outcomes are listed below, along with a representative assessment method for each outcome. Student learning outcomes are not subject to review, revision or approval by the College Curriculum Committee)

SLO #1 UNDERSTANDING CONCEPTS
Students will explain and demonstrate partial derivatives, multiple integrals and the major theorems of vector calculus.
SLO #2 SOLVING PROBLEMS
Students will calculate partial derivatives for a function of more than one variable and use them to solve multivariable optimization problems; and evaluate double and triple integrals, and apply them to physical problems such as moments and centers of mass.
SLO #3 GRAPHS
Students will analyze the graphs and equations of curves and surfaces in three-dimensional space, as well as vector fields.
SLO #4 PROOFS
Students will analyze and apply Green's, Stokes, and Gauss' Theorems.

B. Course Student Learning Objectives (The major learning objective for students enrolled in this course are listed below.)

1. Use vectors and vector operations in two and three dimensions (addition, scalar multiplication, magnitude, dot-product, cross product, triple product and projections) in a variety of problems.
2. Determine vector, parametric, and symmetric equations of lines; vector, parametric, and rectangular equations of planes and quadric surfaces.
3. Find tangent, normal and binormal vectors, arc length, curvature, velocity and acceleration.
4. Convert between rectangular, cylindrical and spherical coordinates.
5. Find and sketch level curves and level surfaces.
6. Determine limits and the continuity of functions of several variables, and prove the existence or non-existence of limits.
7. Calculate partial derivatives, check differentiability, and use the chain rule to find partial derivatives, total derivatives, and higher order derivatives of functions of severable variables.
8. Find differentials and the equations of tangent planes and use in applications.
9. Find gradient vector fields and directional derivatives, and use in applications.
10. Determine extreme values and saddle points of functions of several variables with and without Lagrange multipliers and use in applications.
11. Determine if vector fields are conservative and find potential functions.
12. Evaluate multiple integrals directly and by converting to cylindrical or spherical coordinates.
13. Use multiple integrals to find plane areas, surface areas, and volumes.
14. Evaluate line and surface integrals including parametrically defined surfaces, and find flux.
15. Calculate the curl and divergence of vector fields and use these to solve problems.
16. Use Green's Theorem, Stokes' Theorem and Gauss' Theorem to solve a variety of problems.

III. OUTLINE OF SUBJECT MATTER (Topics are detailed enough to enable a qualified instructor to determine the major areas that should be covered as well as ensure consistency from instructor to instructor and semester to semester.)

Lecture or Lab	Approximate Hours	Topic Number	Major Topic
Lecture	22	I	TWO and THREE DIMENSIONAL ANALYTIC GEOMETRY AND VECTORS A. Vectors and vector operations, dot-product and cross-product B. Equations of lines, planes and quadric surfaces C. Triple products and projections D. Applications of vectors (distances, volumes, etc.) E. Vector-valued functions and space curves F. Arc length and curvature G. Velocity and acceleration H. Cylindrical and spherical coordinates
Lecture	22	II	FUNCTIONS OF SEVERAL VARIABLES A. Limits and continuity B. Partial derivatives and differentiability C. Tangent planes and linear approximations D. Chain rule and higher order derivatives E. Directional derivatives and gradient F. Extrema, saddle points, and Lagrange multipliers G. Applications
Lecture	18	III	MULTIPLE INTEGRALS A. Double integrals in rectangular and polar coordinates B. Surface area C. Triple integrals in rectangular, cylindrical and spherical coordinates D. Change of variables and Jacobian
Lecture	28	IV	VECTOR CALCULUS A. Vector fields including gradient vector fields and conservative vector fields B. Line integrals and the Fundamental Theorem for Line Integrals C. Green's Theorem D. Curl and divergence E. Surface integrals including parametrically defined surfaces F. Stokes' and Gauss' Theorems and their applications
Total Lecture Hours		90	
Total Laboratory Hours		0	
Total Hours		90	

IV. PRIMARY METHOD OF EVALUATION AND SAMPLE ASSIGNMENTS

A. PRIMARY METHOD OF EVALUATION:

Problem solving demonstrations (computational or non-computational)

B. TYPICAL ASSIGNMENT USING PRIMARY METHOD OF EVALUATION:

Read the section on Partial Derivatives and do the assigned problems, including, for example:

Given $z = \ln(\sin(x-y))$ find $\partial^3 z/\partial y \partial x^2$

C. COLLEGE-LEVEL CRITICAL THINKING ASSIGNMENTS:

1. A cardboard box without a lid is to have a volume of 32,000 cm^3. Find the dimensions that minimize the cost of the materials.

   For a box with the same volume as above, how would your answer (i.e. the dimensions) change if the cost of the material used for the bottom of the box was twice as much as that used for the side?

2. Use a triple integral to find the volume of the solid bounded by the elliptic cylinder $4x^2 + z^2 = 4$ and the planes $y=0$ and $y = z + 2$. Sketch the volume described.

C. OTHER TYPICAL ASSESSMENT AND EVALUATION METHODS:

Objective Exams
Other exams
Quizzes
Homework Problems

V. INSTRUCTIONAL METHODS

Discussion
Lecture

Note: In compliance with Board Policies 1600 and 3410, Title 5 California Code of Regulations, the Rehabilitation Act of 1973, and Sections 504 and 508 of the Americans with Disabilities Act, instruction delivery shall provide access, full inclusion, and effective communication for students with disabilities.

VI. WORK OUTSIDE OF CLASS

Study
Required reading
Problem solving activities

Estimated Independent Study Hours per Week: 10

VII. TEXTS AND MATERIALS

A. UP-TO-DATE REPRESENTATIVE TEXTBOOKS

Stewart, James, et al. CALCULUS: EARLY TRANSCENDENTALS, 9th Edition. Cengage, 2021.

B. ALTERNATIVE TEXTBOOKS

C. REQUIRED SUPPLEMENTARY READINGS

D. OTHER REQUIRED MATERIALS

VIII. CONDITIONS OF ENROLLMENT

A. Requisites (Course and Non-Course Prerequisites and Corequisites)

Requisites	Category and Justification
Course Prerequisite Mathematics-191	Sequential

B. Requisite Skills

Requisite Skills
Differentiate functions of a single variable including the use of the product rule, the quotient rule, the chain rule, and the definition of the derivative. MATH 191 - Solve problems using Taylor series, including differentiation and integration of power series. MATH 191 - Solve problems involving parametric equations, polar coordinates and conic sections. Examples include the graphing of parametric and polar curves and the calculation of the arc length of curves so defined. Additional problems involve the calculation of the area bounded by such curves.
Use differentiation in applications MATH 191 - Evaluate integrals using integration techniques including: integration by parts; trigonometric substitutions; partial fraction decomposition and tables of integrals. MATH 191 - Solve problems using Taylor series, including differentiation and integration of power series.
Integrate functions of a single variable using a variety of techniques. MATH 191 - Evaluate integrals using integration techniques including: integration by parts; trigonometric substitutions; partial fraction decomposition and tables of integrals. MATH 191 -Use numerical techniques (both with and without technology) to approximate the values of integrals.
Use integration in applications. MATH 191 -Use integration to solve application problems involving: areas between curves; volumes of solids of known cross section; volumes of solids of revolution; work; arc length and areas of surfaces of revolution. MATH 191 -Solve problems involving parametric equations, polar coordinates and conic sections. Examples include the graphing of parametric and polar curves and the calculation of the arc length of curves so defined. Additional problems involve the calculation of the area bounded by such curves.

C. Recommended Preparations (Course and Non-Course)

Recommended Preparation	Category and Justification

D. Recommended Skills

Recommended Skills

E. Enrollment Limitations

Enrollment Limitations and Category	Enrollment Limitations Impact

Course created by H. Jones on 01/01/1967.

BOARD APPROVAL DATE:

LAST BOARD APPROVAL DATE: 12/21/2020

Last Reviewed and/or Revised by: Paul Yun on 10/10/2020
18683